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Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Further Mathematics

Advanced Subsidiary
Further Mathematics options
Paper 2B: Further Pure Mathematics 1 and Further Statistics 1

Sample Assessment Material for first teaching September 2017
Time: 1 hour 40 minutes

Paper Reference
8FM0/2B

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 80.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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SECTION A

Answer ALL questions. Write your answers in the spaces provided.

1. (a) Use the substitution $t = \tan\left(\frac{x}{2}\right)$ to show that

$$\sec x - \tan x \equiv \frac{1-t}{1+t} \qquad x \neq (2n+1)\frac{\pi}{2},\ n \in \mathbb{Z}$$

(3)

(b) Use the substitution $t = \tan\left(\frac{x}{2}\right)$ and the answer to part (a) to prove that

$$\frac{1-\sin x}{1+\sin x} \equiv (\sec x - \tan x)^2 \qquad x \neq (2n+1)\frac{\pi}{2},\ n \in \mathbb{Z}$$

(3)

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Question 1 continued

(Total for Question 1 is 6 marks)

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2. The value, V hundred pounds, of a particular stock t hours after the opening of trading on a given day is modelled by the differential equation

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{V^2 - t}{t^2 + tV} \qquad 0 < t < 8.5$$

A trader purchases £300 of the stock one hour after the opening of trading.

Use two iterations of the approximation formula $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0 \approx \frac{y_1 - y_0}{h}$ to estimate, to the nearest £, the value of the trader's stock half an hour after it was purchased.

(6)

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Question 2 continued

(Total for Question 2 is 6 marks)

3. Use algebra to find the set of values of x for which

$$\frac{1}{x} < \frac{x}{x+2}$$

(6)

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Question 3 continued

(Total for Question 3 is 6 marks)

4.

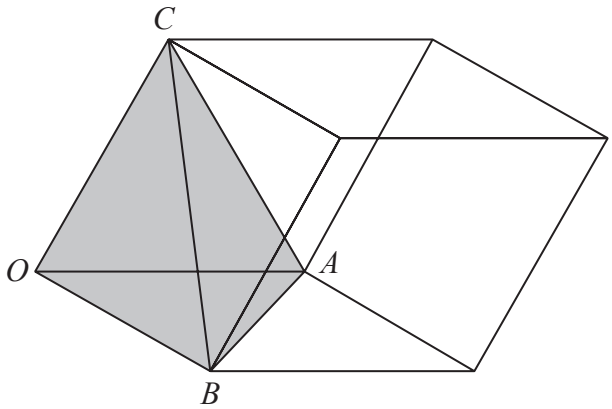

Figure 1

Figure 1 shows a sketch of a solid sculpture made of glass and concrete. The sculpture is modelled as a parallelepiped.

The sculpture is made up of a concrete solid in the shape of a tetrahedron, shown shaded in Figure 1, whose vertices are $O(0, 0, 0)$, $A(2, 0, 0)$, $B(0, 3, 1)$ and $C(1, 1, 2)$, where the units are in metres. The rest of the solid parallelepiped is made of glass which is glued to the concrete tetrahedron.

(a) Find the surface area of the glued face of the tetrahedron.

(4)

(b) Find the volume of glass contained in this parallelepiped.

(5)

(c) Give a reason why the volume of concrete predicted by this model may not be an accurate value for the volume of concrete that was used to make the sculpture.

(1)

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Question 4 continued

(Total for Question 4 is 10 marks)

5.

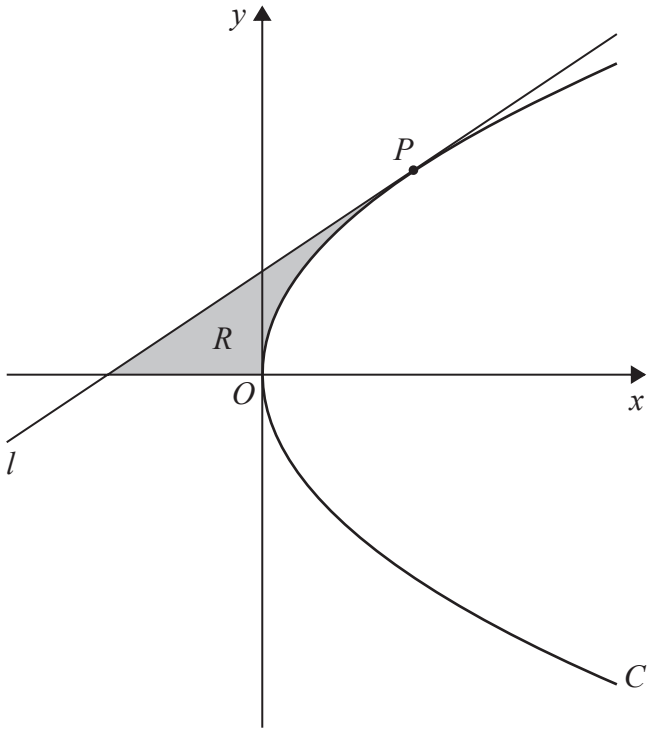

Diagram not drawn to scale

Figure 2

$$\left[\text{You may quote without proof that for the general parabola } y^2 = 4ax,\ \frac{dy}{dx} = \frac{2a}{y}\right]$$

The parabola C has equation $y^2 = 16x$.

(a) Deduce that the point $P(4p^2, 8p)$ is a general point on C.

(1)

The line l is the tangent to C at the point P.

(b) Show that an equation for l is

$$py = x + 4p^2$$

(3)

The finite region R, shown shaded in Figure 2, is bounded by the line l, the x-axis and the parabola C.

The line l intersects the directrix of C at the point B, where the y coordinate of B is $\frac{10}{3}$

Given that $p > 0$

(c) show that the area of R is 36

(8)

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Question 5 continued

Question 5 continued

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Question 5 continued

(Total for Question 5 is 12 marks)

TOTAL FOR SECTION A IS 40 MARKS

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SECTION B

Answer ALL questions. Write your answers in the spaces provided.

6. A university foreign language department carried out a survey of prospective students to find out which of three languages they were most interested in studying.

A random sample of 150 prospective students gave the following results.

		Language		
		French	**Spanish**	**Mandarin**
Gender	**Male**	23	22	20
	Female	38	32	15

A test is carried out at the 1% level of significance to determine whether or not there is an association between gender and choice of language.

(a) State the null hypothesis for this test.

(1)

(b) Show that the expected frequency for females choosing Spanish is 30.6

(1)

(c) Calculate the test statistic for this test, stating the expected frequencies you have used.

(3)

(d) State whether or not the null hypothesis is rejected. Justify your answer.

(2)

(e) Explain whether or not the null hypothesis would be rejected if the test was carried out at the 10% level of significance.

(1)

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Question 6 continued

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Question 6 continued

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Question 6 continued

(Total for Question 6 is 8 marks)

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7. The discrete random variable X has probability distribution given by

x	-1	0	1	2	3
$P(X = x)$	c	a	a	b	c

The random variable $Y = 2 - 5X$

Given that $E(Y) = -4$ and $P(Y \geqslant -3) = 0.45$

(a) find the probability distribution of X.

(7)

Given also that $E(Y^2) = 75$

(b) find the exact value of $\text{Var}(X)$

(2)

(c) Find $P(Y > X)$

(2)

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Question 7 continued

Question 7 continued

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Question 7 continued

(Total for Question 7 is 11 marks)

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8. Two car hire companies hire cars independently of each other.

Car Hire A hires cars at a rate of 2.6 cars per hour.

Car Hire B hires cars at a rate of 1.2 cars per hour.

(a) In a 1 hour period, find the probability that each company hires exactly 2 cars.

(2)

(b) In a 1 hour period, find the probability that the total number of cars hired by the two companies is 3

(2)

(c) In a 2 hour period, find the probability that the total number of cars hired by the two companies is less than 9

(2)

On average, 1 in 250 new cars produced at a factory has a defect.

In a random sample of 600 new cars produced at the factory,

(d) (i) find the mean of the number of cars with a defect,

(ii) find the variance of the number of cars with a defect.

(2)

(e) (i) Use a Poisson approximation to find the probability that no more than 4 of the cars in the sample have a defect.

(ii) Give a reason to support the use of a Poisson approximation.

(2)

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Question 8 continued

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Question 8 continued

(Total for Question 8 is 10 marks)

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9. The discrete random variable X follows a Poisson distribution with mean 1.4

(a) Write down the value of

(i) $P(X = 1)$

(ii) $P(X \leqslant 4)$

(2)

The manager of a bank recorded the number of mortgages approved each week over a 40 week period.

Number of mortgages approved	0	1	2	3	4	5	6
Frequency	10	16	7	4	2	0	1

(b) Show that the mean number of mortgages approved over the 40 week period is 1.4

(1)

The bank manager believes that the Poisson distribution may be a good model for the number of mortgages approved each week.

She uses a Poisson distribution with a mean of 1.4 to calculate expected frequencies as follows.

Number of mortgages approved	0	1	2	3	4	5 or more
Expected frequency	9.86	r	9.67	4.51	1.58	s

(c) Find the value of r and the value of s giving your answers to 2 decimal places.

(2)

The bank manager will test, at the 5% level of significance, whether or not the data can be modelled by a Poisson distribution.

(d) Calculate the test statistic and state the conclusion for this test. State clearly the degrees of freedom and the hypotheses used in the test.

(6)

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Question 9 continued

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Question 9 continued

(Total for Question 9 is 11 marks)

TOTAL FOR SECTION B IS 40 MARKS
TOTAL FOR PAPER IS 80 MARKS

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Paper 2 Option B

Further Pure Mathematics 1 Mark Scheme (Section A)

Question	Scheme	Marks	AOs
1(a)	$\sec x - \tan x = \dfrac{1}{\frac{1-t^2}{1+t^2}} - \dfrac{2t}{1-t^2}$	M1	2.1
	$= \dfrac{1+t^2}{1-t^2} - \dfrac{2t}{1-t^2} = \dfrac{1-2t+t^2}{1-t^2}$	M1	1.1b
	$= \dfrac{(1-t)^2}{(1-t)(1+t)} = \dfrac{1-t}{1+t}$ *	A1*	2.1
		(3)	
(b)	$\dfrac{1-\sin x}{1+\sin x} = \dfrac{1-\frac{2t}{1+t^2}}{1+\frac{2t}{1+t^2}}$	M1	1.1a
	$= \dfrac{1+t^2-2t}{1+t^2+2t}$	M1	1.1b
	$= \dfrac{(1-t)^2}{(1+t)^2} = \left(\dfrac{1-t}{1+t}\right)^2 = (\sec x - \tan x)^2$ *	A1*	2.1
		(3)	
			(6 marks)

Notes:

(a)

M1: Uses $\sec x = \dfrac{1}{\cos x}$ and the t-substitutions for both $\cos x$ and $\tan x$ to obtain an expression in terms of t

M1: Sorts out the $\sec x$ term, and puts over a common denominator of $1-t^2$

A1*: Factorises both numerator and denominator (must be seen) and cancels the $(1+t)$ term to achieve the answer

(b)

M1: Uses the t-substitution for $\sin x$ in both numerator and denominator

M1: Multiples through by $1+t^2$ in numerator and denominator

A1*: Factorises both numerator and denominator and makes the connection with part (a) to achieve the given result

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Question	Scheme	Marks	AOs
2	£300 purchased one hour after opening $\Rightarrow V_0 = 3$ and $t_0 = 1$; half an hour after purchase $\Rightarrow t_2 = 1.5$, so step h required is 0.25	B1	3.3
	$t_0 = 1,\ V_0 = 3,\ \left(\frac{dV}{dt}\right)_0 \approx \frac{3^2 - 1}{1^2 + 3} = 2$	M1	3.4
	$V_1 \approx V_0 + h\left(\frac{dV}{dt}\right)_0 = 3 + 0.25 \times 2 = \ldots$	M1	1.1b
	$= 3.5$	A1ft	1.1b
	$\left(\frac{dV}{dt}\right)_1 \approx \frac{3.5^2 - 1.25}{1.25^2 + 1.25 \times 3.5} \left(= \frac{176}{95}\right)$	M1	1.1b
	$V_2 \approx V_1 + h\left(\frac{dV}{dt}\right)_1 = 3.5 + 0.25 \times \frac{176}{95} = 3.963\ldots$, so £396 (nearest £)	A1	3.2a
		(6)	
			(6 marks)

Notes:

B1: Identifies the correct initial conditions and requirement for h

M1: Uses the model to evaluate $\frac{dV}{dt}$ at t_0, using their t_0 and V_0

M1: Applies the approximation formula with their values

A1ft: 3.5 or exact equivalent. Follow through their step value

M1: Attempt to find $\left(\frac{dV}{dt}\right)_1$ with their 3.5

A1: Applies the approximation and interprets the result to give £396

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Question	Scheme	Marks	AOs
3	$\frac{1}{x} < \frac{x}{x+2}$		
	$\frac{(x+2)-x^2}{x(x+2)} < 0$ or $x(x+2)^2 - x^3(x+2) < 0$	M1	2.1
	$\frac{x^2-x-2}{x(x+2)} > 0 \Rightarrow \frac{(x-2)(x+1)}{x(x+2)} > 0$ or $x(x+2)(2-x)(x+1) < 0$	M1	1.1b
	At least two correct critical values from $-2, -1, 0, 2$	A1	1.1b
	All four correct critical values $-2, -1, 0, 2$	A1	1.1b
	$\{x \in \mathbb{R} : x < -2\} \cup \{x \in \mathbb{R} : -1 < x < 0\} \cup \{x \in \mathbb{R} : x > 2\}$	M1 A1	2.2a 2.5
		(6)	
			(6 marks)

Notes:

M1: Gathers terms on one side and puts over common denominator, or multiply by $x^2(x+2)^2$ and then gather terms on one side

M1: Factorise numerator or find roots of numerator or factorise resulting in equation into 4 factors

A1: At least 2 correct critical values found

A1: Exactly 4 correct critical values

M1: Deduces that the 2 "outsides" and the "middle interval" are required. May be by sketch, number line or any other means

A1: Exactly 3 correct intervals, accept equivalent set notations, but must be given as a set e.g. accept $\mathbb{R} - ([-2, -1] \cup [0, 2])$ or $\{x \in \mathbb{R} : x < -2 \text{ or } -1 < x < 0 \text{ or } x > 2\}$

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Question	Scheme	Marks	AOs
4(a)	Identifies glued face is triangle ABC and attempts to find the area, e.g. evidences by use of $\frac{1}{2}\lvert\mathbf{AB}\times\mathbf{AC}\rvert$	M1	3.1a
	$\frac{1}{2}\lvert\mathbf{AB}\times\mathbf{AC}\rvert = \frac{1}{2}\lvert(-2\mathbf{i}+3\mathbf{j}+\mathbf{k})\times(-\mathbf{i}+\mathbf{j}+2\mathbf{k})\rvert$	M1	1.1b
	$=\frac{1}{2}\lvert 5\mathbf{i}+3\mathbf{j}+\mathbf{k}\rvert$	M1	1.1b
	$=\frac{1}{2}\sqrt{35}\,(\text{m}^2)$	A1	1.1b
		(4)	
	Alternative		
	Identifies glued face is triangle ABC and attempts to find the area, e.g. evidences by use of $\frac{1}{2}\sqrt{\lvert\mathbf{AB}\rvert^2\lvert\mathbf{AC}\rvert^2-(\mathbf{AB}.\mathbf{AC})^2}$	M1	3.1a
	$\lvert\mathbf{AB}\rvert^2=4+9+1=14,\quad \lvert\mathbf{AC}\rvert^2=1+1+4=6$ and $\mathbf{AB}.\mathbf{AC}=2+3+2=7$	M1	1.1b
	So area of glue is $=\frac{1}{2}\sqrt{('14')('6')-('7')^2}$	M1	1.1b
	$=\frac{1}{2}\sqrt{35}\ (\text{m}^2)$	A1	1.1b
		(4)	
(b)	Volume of parallelepiped taken up by concrete is e.g. $\frac{1}{6}(\mathbf{OC}.(\mathbf{OA}\times\mathbf{OB}))$	M1	3.1a
	$=\frac{1}{6}(\mathbf{i}+\mathbf{j}+2\mathbf{k}).(2\mathbf{i}\times(3\mathbf{j}+\mathbf{k}))$	M1	1.1b
	$=\frac{10}{6}=\frac{5}{3}$	A1	1.1b
	Volume of parallelepiped is 6 $\times$ volume of tetrahedron (= 10), so volume of glass is difference between these, viz. $10-\frac{5}{3}=\ldots$	M1	3.1a
	Volume of glass $=\frac{25}{3}(\text{m}^3)$	A1	1.1b
		(5)	

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Question	Scheme	Marks	AOs
	4(b) Alternative		
	$-\mathbf{j}+3\mathbf{k}$ is perpendicular to both $\mathbf{OA} = 2\mathbf{i}$ and $\mathbf{OB} = 3\mathbf{j}+\mathbf{k}$	M1	3.1a
	Area $AOB = \frac{1}{2}\times\lvert\mathbf{OA}\rvert\times\lvert\mathbf{OB}\rvert = \frac{1}{2}\times 2\times\sqrt{10} = \sqrt{10}$	A1	1.1b
	$\mathbf{i}+\mathbf{j}+2\mathbf{k}-p(-\mathbf{j}+3\mathbf{k}) = \mu(2\mathbf{i})+\lambda(3\mathbf{j}+\mathbf{k}) \Rightarrow p = \frac{1}{2}$ and so height of tetrahedron is $h = \frac{1}{2}\lvert-\mathbf{j}+3\mathbf{k}\rvert = \frac{1}{2}\sqrt{10}$	M1	3.1a
	Volume of glass is $V = 5 \times$ Volume of tetrahedron $= 5\times\frac{1}{3}\sqrt{10}\times\frac{1}{2}\sqrt{10}$	M1	1.1b
	$= \frac{25}{3}\left(\text{m}^3\right)$	A1	1.1b
		(5)	
(c)	The glued surfaces may distort the shapes / reduce the volume of concrete Measurements in m may not be accurate The surface of the concrete tetrahedron may not be smooth Pockets of air may form when the concrete is being poured	B1	3.2b
		(1)	
			(10 marks)

Question 4 notes:

Accept use of column vectors throughout

(a)

M1: Shows an understanding of what is required via an attempt at finding the area of triangle *ABC*

M1: Any correct method for the triangle area is fine

M1: Finds **AB** and **AC** or any other appropriate pair of vectors to use in the vector product and attempts to use them

A1: Correct procedure for the vector product with at least 1 correct term $\frac{1}{2}\sqrt{35}$ or exact equivalent

(a) Alternative

M1: Finds two appropriate sides and attempts the scalar product and magnitudes of two of the sides

M1: May use different sides to those shown

M1: Correct full method to find the area of the triangle using their two sides

A1: $\frac{1}{2}\sqrt{35}$ or exact equivalent

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Question 4 notes continued:	
(b)	
M1:	Attempts volume of concrete by finding volume of tetrahedron with appropriate method
M1:	Uses the formula with correct set of vectors substituted (may not be the ones shown) and vector product attempted
A1:	Correct value for the volume of concrete
M1:	Attempt to find total volume of glass by multiplying their volume of concrete by 6 and subtracting their volume of concrete. May restart to find the volume of parallelepiped
A1:	$\frac{25}{3}$ only, ignore reference to units
(b)	**Alternative**
M1:	Notes (or works out using scalar products) that $-\mathbf{j}+\mathbf{3k}$ is a vector perpendicular to both $\mathbf{OA} = 2\mathbf{i}$ and $\mathbf{OB} = 3\mathbf{j}+\mathbf{k}$
A1:	Finds (using that **OA** and **OB** are perpendicular), area of $AOB = \sqrt{10}$
M1:	Solves $\mathbf{i}+\mathbf{j}+\mathbf{2k} - p(-\mathbf{j}+\mathbf{3k}) = \mu(2\mathbf{i})+\lambda(3\mathbf{j}+\mathbf{k})$ to get the height of the tetrahedron $\left[(\mu=\lambda=)\ p=\frac{1}{2},\ \text{so } h=\frac{1}{2}\lvert-\mathbf{j}+3\mathbf{k}\rvert=\frac{1}{2}\sqrt{10}\right]$
M1:	Identifies the correct area as 5 times the volume of the tetrahedron (may be done as in main scheme via the difference)
A1:	$\frac{25}{3}$ only, ignore reference to units
(c)	
B1:	Any acceptable reason in context

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Question	Scheme	Marks	AOs
5(a)	$y^2 = (8p)^2 = 64p^2$ and $16x = 16(4p^2) = 64p^2$ $\Rightarrow P(4p^2, 8p)$ is a general point on C	B1	2.2a
		(1)	
(b)	$y^2 = 16x$ gives $a = 4$, or $2y\frac{dy}{dx} = 16$ so $\frac{dy}{dx} = \frac{8}{y}$	M1	2.2a
	$l: y - 8p = \left(\frac{8}{8p}\right)\left(x - 4p^2\right)$	M1	1.1b
	leading to $py = x + 4p^2$ *	A1*	2.1
		(3)	
(c)	$B\left(-4, \frac{10}{3}\right)$ into $l \Rightarrow \frac{10p}{3} = -4 + 4p^2$	M1	3.1a
	$6p^2 - 5p - 6 = 0 \Rightarrow (2p-3)(3p+2) = 0 \Rightarrow p = ...$	M1	1.1b
	$p = \frac{3}{2}$ and l cuts x-axis when $\frac{3}{2}(0) = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = ...$	M1	2.1
	$x = -9$	A1	1.1b
	$p = \frac{3}{2} \Rightarrow P(9, 12) \Rightarrow \text{Area}(R) = \frac{1}{2}(9 - -9)(12) - \int_0^9 4x^{\frac{1}{2}}dx$	M1	2.1
	$\int 4x^{\frac{1}{2}}dx = \frac{4x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}(+c)$ or $\frac{8}{3}x^{\frac{3}{2}}(+c)$	M1	1.1b
		A1	1.1b
	$\text{Area}(R) = \frac{1}{2}(18)(12) - \frac{8}{3}\left(9^{\frac{3}{2}} - 0\right) = 108 - 72 = 36$ *	A1*	1.1b
		(8)	

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Question	Scheme	Marks	AOs
	5(c) Alternative 1		
	$B\left(-4, \frac{10}{3}\right)$ into $l \Rightarrow \frac{10p}{3} = -4 + 4p^2$	M1	3.1a
	$6p^2 - 5p - 6 = 0 \Rightarrow (2p-3)(3p+2) = 0 \Rightarrow p = ...$	M1	1.1b
	$p = \frac{3}{2}$ into l gives $\frac{3}{2}y = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = ...$	M1	2.1
	$x = \frac{3}{2}y - 9$	A1	1.1b
	$p = \frac{3}{2} \Rightarrow P(9, 12) \Rightarrow \text{Area}(R) = \int_0^{12}\left(\frac{1}{16}y^2 - \left(\frac{3}{2}y - 9\right)\right)\mathrm{d}y$	M1	2.1
	$\int\left(\frac{1}{16}y^2 - \frac{3}{2}y + 9\right)\mathrm{d}y = \frac{1}{48}y^3 - \frac{3}{4}y^2 + 9y \ (+c)$	M1 A1	1.1b 1.1b
	$\text{Area}(R) = \left(\frac{1}{48}(12)^3 - \frac{3}{4}(12)^2 + 9(12)\right) - (0)$ $= 36 - 108 + 108 = 36$ *	A1*	1.1b
		(8)	
	5(c) Alternative 2		
	$B\left(-4, \frac{10}{3}\right)$ into $l \Rightarrow \frac{10p}{3} = -4 + 4p^2$	M1	3.1a
	$6p^2 - 5p - 6 = 0 \Rightarrow (2p-3)(3p+2) = 0 \Rightarrow p = ...$	M1	1.1b
	$p = \frac{3}{2}$ and l cuts px-axis when $\frac{3}{2}(0) = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = ...$	M1	2.1
	$x = -9$	A1	1.1b
	$p = \frac{3}{2} \Rightarrow P(9, 12)$ and $x = 0$ in $l: y = \frac{2}{3}x + 6$ gives $y = 6$ $\Rightarrow \text{Area}(R) = \frac{1}{2}(9)(6) + \int_0^9\left(\left(\frac{2}{3}x + 6\right) - \left(4x^{\frac{1}{2}}\right)\right)\mathrm{d}x$	M1	2.1
	$\int\left(\frac{2}{3}x + 6 - 4x^{\frac{1}{2}}\right)\mathrm{d}x = \frac{1}{3}x^2 + 6x - \frac{8}{3}x^{\frac{3}{2}} \ (+c)$	M1 A1	1.1b 1.1b
	$\text{Area}(R) = 27 + \left(\left(\frac{1}{3}(9)^2 + 6(9) - \frac{8}{3}\left(9^{\frac{3}{2}}\right)\right) - (0)\right)$ $= 27 + (27 + 54 - 72) = 27 + 9 = 36$ *	A1*	1.1b
		(8)	
			(12 marks)

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Question 5 notes:

(a)

B1: Substitutes $y_P = 8p$ into y^2 to obtain $64p^2$ and substitutes $x_P = 4p^2$ into $16x$ to obtain $64p^2$ and concludes that P lies on C

(b)

M1: Uses the given formula to deduce the derivative. Alternatively, may differentiate using chain rule to deduce it

M1: Applies $y - 8p = m(x - 4p^2)$, with their tangent gradient m, which is in terms of p.

Accept use of $8p = m(4p^2) + c$ with a clear attempt to find c

A1*: Obtains $py = x + 4p^2$ by **cso**

(c)

M1: Substitutes their $x = "-a"$ and $y = \frac{10}{3}$ into l

M1: Obtains a 3 term quadratic and solves (using the usual rules) to give $p = ...$

M1: Substitutes their p (which must be positive) and $y = 0$ into l and solves to give $x = ...$

A1: Finds that l cuts the x-axis at $x = -9$

M1: Fully correct method for finding the area of R

i.e. $\frac{1}{2}(\text{their } x_P - "-9")(\text{their } y_P) - \int_0^{\text{their } x_P} 4x^{\frac{1}{2}}\,\mathrm{d}x$

M1: Integrates $\pm\lambda x^{\frac{1}{2}}$ to give $\pm\mu x^{\frac{3}{2}}$, where $\lambda, \mu \neq 0$

A1: Integrates $4x^{\frac{1}{2}}$ to give $\frac{8}{3}x^{\frac{3}{2}}$, simplified or un-simplified

A1*: Fully correct proof leading to a correct answer of 36

(c) Alternative 1

M1: Substitutes their $x = "-a"$ and $y = \frac{10}{3}$ into l

M1: Obtains a 3 term quadratic and solves (using the usual rules) to give $p = ...$

Substitutes their p (which must be positive) into l and rearranges to give $x = ...$

M1: Finds l as $x = \frac{3}{2}y - 9$

A1: Fully correct method for finding the area of R

M1: i.e. $\int_0^{\text{their } y_P} \left(\frac{1}{16}y^2 - \text{their}\left(\frac{3}{2}y - 9\right)\right)\mathrm{d}y$

M1: Integrates $\pm\lambda y^2 \pm \mu y \pm \nu$ to give $\pm\alpha y^3 \pm \beta y^2 \pm \nu y$, where $\lambda, \mu, \nu, \alpha, \beta \neq 0$

A1: Integrates $\frac{1}{16}y^2 - \left(\frac{3}{2}y - 9\right)$ to give $\frac{1}{48}y^3 - \frac{3}{4}y^2 + 9y$, simplified or un-simplified

A1*: Fully correct proof leading to a correct answer of 36

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Question 5 notes continued:
(c) Alternative 2
M1: Substitutes their $x = "-a"$ and $y = \frac{10}{3}$ into l
M1: Obtains a 3 term quadratic and solves (using the usual rules) to give $p = ...$
M1: Substitutes their p (which must be positive) and $y = 0$ into l and solves to give $x = ...$
A1: Finds that l cuts the x-axis at $x = -9$
M1: Fully correct method for finding the area of R
i.e. $\frac{1}{2}(\text{their } 9)(\text{their } 6) + \int_0^{\text{their } x_P} \left(\text{their } \left(\frac{2}{3}x + 6\right) - \left(4x^{\frac{1}{2}}\right)\right) \mathrm{d}y$
M1: Integrates $\pm\lambda x \pm \mu \pm \nu x^{\frac{1}{2}}$ to give $\pm\alpha x^2 \pm \mu x \pm \beta x^{\frac{3}{2}}$, where $\lambda, \mu, \nu, \alpha, \beta \neq 0$
A1: Integrates $\left(\frac{2}{3}x + 6\right) - \left(4x^{\frac{1}{2}}\right)$ to give $\frac{1}{3}x^2 + 6x - \frac{8}{3}x^{\frac{3}{2}}$, simplified or un-simplified
A1*: Fully correct proof leading to a correct answer of 36

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Further Statistics 1 Mark Scheme (Section B)

Question	Scheme	Marks	AOs
6(a)	H_0: There is no association between language and gender	B1	1.2
		(1)	
(b)	$\frac{54\times 85}{150} = 30.6$ *	B1*cso	1.1b
		(1)	
(c)	Expected frequencies: Language – French, Spanish, Mandarin; Gender – Male: 26.43… 23.4 15.16…; Female: 34.56… [30.6] 19.83…	M1	2.1
	$\chi^2 = \sum \frac{(O-E)^2}{E} = \frac{(23-26.43)^2}{26.43} + \ldots + \frac{(15-19.83)^2}{19.83}$	M1	1.1b
	Awrt **3.6/3.7**	A1	1.1b
		(3)	
(d)	Degrees of freedom (3 – 1)(2 – 1) → Critical value $\chi^2_{2,0.01} = 9.210$	M1	3.1b
	As $\sum \frac{(O-E)^2}{E} < 9.210$, the null hypothesis is not rejected	A1	2.2b
		(2)	
(e)	Still not rejected since $\sum \frac{(O-E)^2}{E} < \chi^2_{2,0.1} = 4.605$	B1	2.4
		(1)	
			(8 marks)

Notes:

(a)
B1: For correct hypothesis in context

(b)
B1*: For a correct calculation leading to the given answer and no errors seen

(c)
M1: For attempt at $\frac{\text{(Row Total)(Column Total)}}{\text{(Grand Total)}}$ to find expected frequencies
M1: For applying $\sum \frac{(O-E)^2}{E}$
A1: awrt 3.6 or 3.7

(d)
M1: For using degrees of freedom to set up a χ^2 model critical value
A1: For correct comparison and conclusion

(e)
A1ft: For correct conclusion with supporting reason

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Question	Scheme	Marks	AOs
7(a)	$-4 = 2 - 5\mathrm{E}(X)$	M1	3.1a
	$\mathrm{E}(X) = 1.2$		
	$-1\times c + 0\times a + 1\times a + 2\times b + 3\times c = 1.2$	M1	1.1b
	$a + 2b + 2c = 1.2 \quad \boxed{1}$		
	$\mathrm{P}(Y \geqslant -3) = 0.45$ gives $\mathrm{P}(2 - 5X \geqslant -3) = 0.45$ i.e. $\mathrm{P}(X \leqslant 1) = 0.45$	M1	2.1
	$2a + c = 0.45 \quad \boxed{2}$		
	$2a + b + 2c = 1 \quad \boxed{3}$	M1	1.1b
	$\begin{pmatrix} 1 & 2 & 2 \\ 2 & 0 & 1 \\ 2 & 1 & 2 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1.2 \\ 0.45 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1 & 2 & -2 \\ 2 & 2 & -3 \\ -2 & -3 & 4 \end{pmatrix}\begin{pmatrix} 1.2 \\ 0.45 \\ 1 \end{pmatrix}$ <u>or</u>	M1	1.1b
	e.g. $\boxed{3} - \boxed{2} \Rightarrow b + c = 0.55$ sub. $2(b+c)$ into $\boxed{1} \Rightarrow a = 0.1$ etc		
	$a = 0.1 \quad b = 0.3 \quad c = 0.25$	A1 A1	1.1b 1.1b
		(7)	
(b)	$\mathrm{Var}(Y) = 75 - (-4)^2$ <u>or</u> 59	M1	1.1a
	[$\mathrm{Var}(Y) = 5^2\mathrm{Var}(X)$ implies] $\mathrm{Var}(X) = 2.36$	A1	1.2
		(2)	
(c)	$\mathrm{P}(Y > X) = \mathrm{P}(2 - 5X > X) \rightarrow \mathrm{P}(X < \frac{1}{3})$	M1	3.1a
	$\mathrm{P}(X < \frac{1}{3}) = a + c = 0.35$	A1ft	1.1b
		(2)	
			(11 marks)

Notes:

(a)

M1: For using given information to find an expression for $\mathrm{E}(X)$ i.e. use of $\mathrm{E}(Y) = 2 - 5\mathrm{E}(X)$

M1: For use of $\sum x\mathrm{P}(X = x) =$ '1.2'

M1: For use of $\mathrm{P}(Y \geqslant -3) = 0.45$ to set up the argument for solving by forming an equation in a and c

M1: For use of $\sum \mathrm{P}(X = x) = 1$

M1: For solving their 3 linear equations (matrix or elimination)

A1: For any 2 of a, b or c correct

A1: For all 3 correct values

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Question 7 notes continued:
Another method for part (a) is: **M1:** For using given information to find the probability distribution for Y leading to an expression for E(Y) **M1:** For use of $\sum yP(Y = y) = -4$ **M1:** For use of $P(Y \geqslant -3) = 0.45$ to set up the argument for solving by forming an equation in a and c **M1:** For use of $\sum P(Y = y) = 1$ **M1:** For solving their 3 linear equations (matrix or elimination) **A1:** For any 2 of a, b or c correct **A1:** For all 3 correct values
(b) **M1:** For use of $\text{Var}(Y) = E(Y^2) - [E(Y)]^2$ (may be implied by a correct answer) **A1:** For use of $\text{Var}(aX) = a^2\,\text{Var}(X)$ to reach 2.36 or exact equivalent
(c) **M1:** For rearranging to the form $P(X < k)$ **A1ft:** 0.1' + '025' (provided their a and c and their $a + c$ are all probabilities) **Another method for part (c) is:** **M1:** For comparing distribution of X with distribution of Y to identify $X = -1$ and $X = 0$ **A1ft:** '0.1' + '025' (provided their a and c and their $a + c$ are all probabilities)

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Question	Scheme	Marks	AOs
8(a)	$X \sim \text{Po}(2.6)$ $\quad Y \sim \text{Po}(1.2)$		
	P(each hire 2 in 1 hour) $= \text{P}(X=2) \times \text{P}(Y=2) = 0.25104\ldots \times 0.21685\ldots$	M1	3.3
	$= 0.05444\ldots$ awrt **0.0544**	A1	1.1b
		(2)	
(b)	$W = X + Y \rightarrow W \sim \text{Po}(3.8)$	M1	3.4
	$\text{P}(W = 3) = 0.20458\ldots$ awrt **0.205**	A1	1.1b
		(2)	
(c)	$T \sim \text{Po}((2.6+1.2)\times 2)$	M1	3.3
	$\text{P}(T < 9) = 0.64819\ldots$ awrt **0.648**	A1	1.1b
		(2)	
(d)	**(i)** Mean $= np =$ **2.4**	B1	1.1b
	(ii) Variance $= np(1-p) = 2.3904$ awrt **2.39**	B1	1.1b
		(2)	
(e)	**(i)** [$D \sim \text{Po}(2.4)$ $\quad \text{P}(D \leqslant 4)$] $= 0.9041\ldots$ awrt **0.904**	B1	1.1b
	(ii) Since n is large and p is small/mean is approximately equal to variance	B1	2.4
		(2)	
			(10 marks)

Notes:

(a)
M1: For $\text{P}(X=2) \times \text{P}(Y=2)$ from $X \sim \text{Po}(2.6)$ and $Y \sim \text{Po}(1.2)$ i.e. correct models (may be implied by correct answer)
A1: **awrt 0.0544**

(b)
M1: For combining Poisson distributions and use of Po('3.8') (may be implied by correct answer)
A1: **awrt 0.205**

(c)
M1: For setting up a new model and attempting mean of Poisson distribution (may be implied by correct answer)
A1: **awrt 0.648**

(d)(i)
B1: For **2.4**

(d)(ii)
B1: For **awrt 2.39**

(e)(i)
B1: For **awrt 0.904**

(e)(ii)
B1: For a correct explanation to support use of Poisson approximation in this case

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Question	Scheme	Marks	AOs
9(a)	**(i)** P(X = 1) = 0.34523… awrt **0.345**	B1	1.1b
	(ii) P($X \leqslant 4$) = 0.98575… awrt **0.986**	B1	1.1b
		(2)	
(b)	$\frac{(0\times10)+1\times16+2\times7+3\times4+4\times2+(5\times0)+6\times1}{40}=1.4$*	B1*cso	1.1b
		(1)	
(c)	r = 40 × '0.34523…' s = 40 × '1 – 0.986…'	M1	3.4
	r = **13.81** s = **0.57**	A1ft	1.1b
		(2)	
(d)	H_0: The Poisson distribution is a suitable model H_1: The Poisson distribution is not a suitable model	B1	3.4
	[Cells are combined when expected frequencies < 5] So combine the last 3 cells	M1	2.1
	$\chi^2=\sum\frac{(O-E)^2}{E}=\frac{(10-9.86)^2}{9.86}+\ldots+\frac{(7-(4.51+1.58+0.57))^2}{(4.51+1.58+0.57)}$	M1	1.1b
	awrt **1.1**	A1	1.1b
	Degrees of freedom = 4 – 1 – 1 = 2	B1	3.1b
	(Do not reject H_0 since $1.10<\chi^2_{2,(0.05)}=5.991$). The number of mortgages approved each week follows a Poisson distribution	A1	3.5a
		(6)	
			(11 marks)

Notes:

(a)(i)
B1: awrt 0.345

(a)(ii)
B1: awrt 0.986

(b)
B1*: For a fully correct calculation leading to given answer with no errors seen

(c)
M1: For attempt at r or s (may be implied by correct answers)
A1ft: For both values correct (follow through their answers to part (a))

(d)
B1: For both hypotheses correct (lambda should not be defined so correct use of the model)
M1: For understanding the need to combine cells before calculating the test statistic (may be implied)
M1: For attempt to find the test statistic using $\chi^2=\sum\frac{(O-E)^2}{E}$
A1: awrt 1.1
B1: For realising that there are 2 degrees of freedom leading to a critical value of $\chi_2^2(0.05)=5.991$
A1: Concluding that a Poisson model is suitable for the number of mortgages approved each week

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